Round 1

1. What is the smallest positive integer *n* such that $2016n$ is a perfect cube?

2. Chris and Daniel go strawberry picking and pick 100 pounds of strawberries. They are 99% water by weight, but by the time they get home, some of the water has evaporated. If they are now 98% water by weight, how many pounds of water evaporated?

3. Find all two digit numbers with the following criteria: the sum of the digits is 14; after the number is increased by 46, the product of its digits is 6.

Round 2

4. Suppose $log2 = a$ and $log13 = b$. Find $log_5 3.38$ in terms of *a* and *b* ($logn$ is the base 10 logarithm of *n*).

5. You are a detective, and a murder has just occurred. Witnesses report two masked suspects fleeing the crime scene. After investigating possible suspects, you narrow down your search to five suspects: Andrew, Beatrice, Charlie, David, and Elizabeth. You know the following facts:
   a. If either Andrew or Elizabeth are guilty, then Beatrice would be guilty too
   b. If Charlie is innocent, then Beatrice is too
   c. If David is guilty and Andrew is innocent, then Charlie is innocent

   Which two of the suspects committed the crime?

6. If two real numbers x and y satisfy $(x+5)^2 + (y-12)^2 = 14^2$, find the minimum value of $(x^2 + y^2)$.

Round 3

7. Triangle ABC has $AB = 1$, $AC = \sqrt{15}$, and the midpoint of BC is M. If $AM = 2$, what is the area of triangle ABC?

8. A standard deck of playing cards has two jokers, four of each number 1 through 10, and four jacks, queens, kings, and aces. If Alice draws 4 cards from the deck without replacement, what is the probability that she draws two cards of one kind and two cards of another kind?

9. A positive integer is *prime-like* if it is not divisible by 2, 3, or 5. How many prime-like positive integers are there less than 10000?

Comment [1]: please check solution

Round 4

10. Let ABCD be a square with side length 8. Let M be the midpoint of BC and let ω be the circle passing through M, A, and D. Let O be the center of ω, X be the intersection point (besides A) of ω with AB, and Y be the intersection point of OX and AM. If the length of OY can be written in simplest form as $\frac{m}{n}$, compute $m+n$.

11. Bob is playing a game where he is trying to obtain the largest score possible. In the first round, a random number generator picks a number from 0 to 1. Each round, he can either chose to keep the score or discard it and go to the next round. On round n, the random number generator picks a number from 0 to 2^{n-1}. Supposing that Bob plays optimally, what is the expected value of his final score?

Comment [2]: please check solution

12. A jogger and a biker start simultaneously from point A on a circular track and go in the same direction. Both travel at constant speeds, and the biker is faster than the jogger. The biker completes one lap then meets the jogger at point B. When the jogger completes exactly one lap, the biker has completed three laps and has arrived again at point B. What is the ratio of the speed of the biker to the speed of the jogger?

Round 5

13. We want to design a new chess piece, the American, with the property that (i) the American can never attack itself, and (ii) if an American A1 attacks another American A2, then A2 also attacks A1. Let m be the number of squares that an American attacks when placed in the top left corner of an 8 by 8 chessboard. Let n be the maximal number of Americans that can be placed on the 8 by 8 chessboard such that no Americans attack each other, if one American must be in the top left corner. Find the largest possible value of mn.

14. Let r_1, r_2, r_3, r_4 be the roots of $x^4 - 6x^3 + 18x^2 - 27x + 9$. Find the value of $\frac{r_1^2}{r_2^2+r_3^2+r_4^2} + \frac{r_2^2}{r_1^2+r_3^2+r_4^2} + \frac{r_3^2}{r_1^2+r_2^2+r_4^2} + \frac{r_4^2}{r_1^2+r_2^2+r_3^2}$.

15. In quadrilateral ABCD, $\angle DAC \cong \angle DBC$, and the ratio of the area of triangle ADB to the area of triangle ABC is ½. Let O be the intersection of BD and AC. If $AD = 4$, $BC = 6$, $BO = 1$, and the area of the quadrilateral is $\frac{a\sqrt{b}}{c}$ in simplest form. Find $a+b+c$.

Round 6

16. A line with inclination angle 60° is drawn through focus F of parabola $y^2 = 8(x+2)$ and intersects the parabola at points A and B. If the perpendicular bisector of $\bar{AB}$ intersects the x-axis at point P, find PF.

17. There is an alphabet A with 16 letters and a permutation $f : A \to A$. Let $n(f)$ be the smallest positive integer *k* such that every message *m* written in A, encrypted by applying *f* to the message *k* times, produces *m*. Compute the largest possible value of $n(f)$.

18. Let ABC be a triangle with AB = 13, AC = 14, and BC = 15. Let G be the point on AC such that the reflection of BG over the angle bisector of $\angle$B passes through the midpoint of AC. Let Y be the midpoint of GC and X be a point on segment AG such that $\frac{AX}{XG} = 3$. Construct F and H on AB and BC, respectively, such that $FX \parallel BG \parallel HY$. If AH and CF concur at Z and W is on AC such that $WZ \parallel BG$, find W Z.

Round 7

19. Let a and b be complex numbers satisfying the two equations $a^3 - 3ab^2 = 36$, $b^3 - 3ba^2 = 28i$. Let M be the maximum possible magnitude of a. Find all a such that |a| = M.

20. Find all real solutions to $x^3 - 8 = 16\sqrt[3]{x+1}$.

21. A knight is placed at the origin of the Cartesian plane. Each turn, the knight moves in an L-shape (2 units parallel to one axis and 1 unit parallel to the other) to one of eight possible locations, chosen at random. After 2016 such turns, what is the expected value of the square of the distance of the knight from the origin?

Round 8

22. There are 11 mathletes at a party, and each pair of mathletes is either friendly or unfriendly. When 3 pairwise friendly mathletes meet up, they gossip and end up in a fight, but remain friends. When 3 pairwise unfriendly mathletes meet, they fight and remain unfriendly. In all other cases, there are no fights. If all possible triples of mathletes meet exactly once, what is the minimum possible number of fights?

23. Let P and A denote the perimeter and area respectively of a right triangle with relatively prime integer side-lengths. Find the largest possible integral value of $\frac{P^2}{A}$.

24. Suppose $P(x) = x^{2016} + a_{2015}x^{2015} + ... + a_1x + a_0$ satisfies $P(x)P(2x+1) = P(-x)P(-2x-1)$ for all real x. Find the sum of all possible values of a_{2015}.

Grade 7-8 Guts Round Solutions

1. 294
2. 50
3. 77, 86
4. $\frac{2b+a-2}{1-a}$
5. Beatrice and Charlie
6. 1
7. $\sqrt{15}/2$
8. $\frac{146}{8109}$
9. 2666
10. 36
11. $\frac{15}{28}$
12. $2+\sqrt{2}$
13. 1024
14. -4
15. 186
16. 16/3
17. 140
18. $\frac{1170\sqrt{37}}{1379}$
19. $3, -\frac{3}{2} \pm \frac{3i\sqrt{3}}{2}$
20. $-2, 1 \pm \sqrt{5}$
21. 10080
22. 28
23. 45
24. 339024

Comment [3]: Please check solution

Comment [4]: Please check solution

Comment [5]: Yea I got 15/28