GSM
Gauss School of Mathematics

Gauss School and Gauss Math Circle
2015 Gauss Math Tournament
Grade 7-8 (Sprint Round 50 minutes)

1. How many integer solutions are there to the inequality $|x - 6| \leq 4$?

2. Find the 2010th digit after the decimal point in the expansion of 1/7?

3. A tetrahedron is a triangular pyramid. What is the sum of the number of edges, faces, and vertices of a tetrahedron?

4. What is the sum of the 2009 fractions of the form 2/n(n+1) if the values of n are the positive integers from 1 to 2009?

5. If the least common multiple of the first 2006 positive integers is m, and the least common multiple of the first 2007 positive integers is km, what is the value of k?

6. The areas of three of the faces of a right, rectangular prism are 24, 32 and 48. What is the volume of the prism?

7. A running track is the ring formed by two concentric circles. If the circumferences of the two circles differ by 10π feet, how wide is the track, in feet?

8. What is the tenth positive integer that is both odd and a multiple of 3?

9. How many positive three-digit integers less than 500 have at least two digits that are the same?

10. Two distinct positive integers x and y are factors of 36. What is the least product xy that is not a factor of 36?

11. The height of a right cylinder is 2.5 times its radius. If the surface area of the cylinder is 112π , what is the radius of the cylinder in centimeters?

12. In quadrilateral SEAT, SE=2, EA=3, AT=4, angle EAT = angle SET = 90. What is the area of the quadrilateral?

13. What is the number of square units in the area of trapezoid ABCD with vertices A (0, 0), B (0, -2), C (4, 0), D (4, 6)?

14. The volume of a rectangular solid is 250. If its length is tripled, its width is halved, and its height is quadrupled, what is the new volume?

15. When (x+2y)^5 is expanded, what is the sum of the coefficients of the polynomial?

16. The mean and median of a set of three numbers is 16, and the smallest number of the three is 5. What is the largest of the three numbers?

17. The price of a dress was increased by n%, and then later that new price was decreased by n%. The resulting price was 96% of the original price. What is the value of n?

18. Jack rolls two standard, six-sided dice. What is the probability that he rolls a sum that is a prime numbers? Express as a common fraction.

19. The numerical value of the volume of a cube is twice the numerical value of the cube's surface area. What is the length of the edge of the cube?

20. If (7, 9) and (10, 2) are the coordinates of two opposite vertices of a square, what is the sum of the y-coordinates of the other two vertices?

21. A solid 6-by-6-by-6 cube is constructed from unit cubes and the entire outer surface is painted. What fraction of the unit cube has at least two painted faces? Express as a common fraction.

22. For each real number x, let f (x) be the minimum of the numbers 3x+1, 2x+3, -4x+24. What is the maximum value of f (x)?

23. In rectangle $ABCD$, the midpoints of sides BC, CD and DA are P, Q and R respectively. The point M is the midpoint of QR. The area of triangle APM is a fraction m / n of the area of rectangle $ABCD$, where m and n are integers and m / n is in its simplest form. What is the value of $m + n$?

24. The integers a, b and c are such that 0<a<b<c<10. The sum of all three-digit numbers that can be formed by a permutation of these three integers is 1554. What is the value of c?

25. Given that $(a+\frac{1}{a})$^2 =6 and a^3 + 1/a^3 = N$\sqrt{6}$ and a > 0. What is the value of N?

26. A circle with diameter AB. The coordinates of A are $(-2, 0)$ and the coordinates of B are $(8, 0)$. The circle cuts the y-axis at points D and E. What is the length of DE?

27. Rachel draws 36 kangaroos using three different colors. 25 of the kangaroos are drawn using some grey, 28 are drawn using some pink and 20 are drawn using some brown. Five of the kangaroos are drawn using all three colors. How many kangaroos did she draw that use only one color?

28. The equation $x2 - bx + 80 = 0$, where $b > 0$, has two integer-valued solutions. What is the sum of the possible values of b?

29. Given that a+b=5 and ab=3, what is the value of a^4 + b^4?

30. The square $ABCD$ has sides of length 1. All possible squares that share two vertices with $ABCD$ are drawn. The boundary of the region formed by the union of these squares is an irregular polygon. What is the area of this polygon?

31. The increasing sequence 1, 3, 4, 9, 10, 12, 13, ... contains all the powers of 3 and all the numbers that can be written as the sum of two or more distinct powers of 3. What is the 70th number in the sequence?

32. How many subsets containing an odd number of elements does a set with 10 elements have?

33. The solutions x=u and x=v of the quadratic equation rx^2 + sx +t = 0 are reciprocals of the solutions of the quadratic equation $(2 + a)x^2$ + 5x + (2 – a) = 0 for some integer a. If the GCF of r, s and t is 1, what is the value of r + s + t.

34. A box contains only quarters and dimes. If there were 10% more quarters, the total values of the money in the box would increase by 7.5%. What is the ratio of the number of quarters to the number of dimes in the box? Express your answer as a common fraction.

35. Rhombus EFGH is inscribed in rhombus ABCD with point E on AB, point F on BC, point G on CD and point H on AD. If AE:EB = BF:FC = CG:GD = DH:HA = 1:2, and if the area of rhombus ABCD is 180, what is the area of rhombus EFGH?

36. AC is the diameter of the circle. If the measure of arc (BC)=60 degree, AC is perpendicular to BE and BE=12 units, what is the length of AB?

37. A coin of radius 1cm is tossed onto a plane surface that has been tiled by equilateral triangles with side length $20\sqrt{3}$ cm. What is the probability that the coin lands within one of the triangles?

38. Find the sum of all positive four-digit numbers that are perfect squares and that have remainder 1 when divided by 100.

39. Three numbers are randomly chosen without replacement from the set (101, 102, 103, ….,, 200). What is the probability that these three numbers are the side lengths of a triangle?

40. In the quadrilateral PEAR, PE=21, EA=20, AR=15, RE=25 and AP=29. Find the area of the quadrilateral?

<u>Sprint Round Ends</u>

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1. A very large lucky number N contains of eighty-eight 8s in a row. Find the remainder when N is divided by 6.

2. Quadrilateral ABCD has vertices at points A (-9, 4), B (-7, 8), C (-3, 6) and D (-6, 2). Quadrilateral WXYZ is congruent to quadrilateral ABCD and has vertices W (2, -3), X (4, 1), and Y (8, -1) and a fourth vertex Z. What is the sum of the coordinates of vertex Z?

3. A right triangle has integer side lengths a, b and c with $a<b<c$. If a + c = 49, what is the area of the triangle?

4. The graphs of y = x^2 - 8x – 35 and y = -2x^2 + 16x + 3 intersect in two points. What is the sum of the x-coordinates of the two points of intersection?

5. Two different unit squares are randomly selected from the 16-unit square in the 4*4 grid shown. What is the probability that they do not have a vertex in common? Express your answer as a common fraction.

6. Four congruent semicircles are drawn within the boundary of a square with side length 1. The center of each semicircle is the midpoint of a side of the square. Each semicircle is tangent to two other semicircles. Region R consists of points lying inside the square but outside of the semicircles. The area of R can be written in the form a – bπ, where a and b are positive rational numbers. Compute a+b.

7. Let x and y be two numbers satisfying the relations x $\geq$ 0, y $\geq$ 0 and 3x + 5y = 7. What is the maximum possible value of 9x^2+ 25y^2?

8. How many ordered pairs (x, y) of integers are there such that 2xy + x + y = 52?

Target Round Ends

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Name: ____________________________ Grade:________________________________

Sprint Round Answers:

1		21	
2		22	
3		23	
4		24	
5		25	
6		26	
7		27	
8		28	
9		29	
10		30	
11		31	
12		32	
13		33	
14		34	
15		35	
16		36	
17		37	
18		38	
19		39	
20		40	

Target Round Answers:

1		5	
2		6	
3		7	
4		8	