Gauss School and Gauss Math Circle
2017 Gauss Math Tournament
<u>Grade 5-6 (Sprint Round 50 minutes)</u>

1. What is (10008)(9992) - (10007)(9993)?
2. What is the 3rd largest number that can be made by using each of the digits 1, 2, 4, 6, and 7 once?
3. Out of 20 students, 14 like pans, 5 like both pots and pans, and 1 likes neither pots nor pans. How many students like pots?
4. Caroline has 16 crayons: 3 yellow, 5 red, 6 orange, and 2 blue. She blindly chooses crayons without replacement. Given that she first chooses a yellow crayon, what is the probability that the second crayon she chooses is orange?
5. What is the sum of all of the factors of 45?
6. What is the sum of the external angles of a hexagon?
7. Arthur Arnolds is at the grocery store. He wants to buy a certain number of pickles and cucumbers. At the store, there are a total of 8 different jars of pickles and 5 different cucumbers. He wishes to buy 3 jars of pickles and 2 cucumbers. How many distinct purchases can Arthur make?
8. A circle with center at point A is externally tangent to another circle with center at point B at point H. If Circle A has a radius of 3, and Circle B has a radius of 5, what is the length of a segment that is externally tangent to both circles?
9. Solve for y given the following system of equations:
$$3y + 19z = 25$$
$$x - 3y - 15z = 10$$
$$x - y + 4z = 4$$
10. How many factors does the number 12408 have?
11. Jane has a bag of 15 marbles: 5 green, 7 red, and 3 blue. She blindly reaches into the bag to pick marbles. How many marbles must she pick to ensure that she has at least 3 marbles of the same color?
12. How many ways are there to make three-letter "words" out of the letters in COMBINE?
13. What is the sum of the internal angles of a 5-pointed star in degrees?
14. Let n be a positive integer. When you divide n by 12, the remainder is 7. What is the remainder when you divide (13n + 2) by 12?
15. Two chords of a circle, EZ and MN intersect at point P inside the circle. EP = 5, ZP = 4, and MP = 2. What is NP?
16. A cube's surface area is numerically equal to 10 times its volume. What is the area of one side of the cube?
17. If the length of a rectangle is increased by 20% and the width decreased by 30%, what is the ratio of the area of the new rectangle to the area of the old rectangle?
18. How many paths of length 8 are there from (0, 0) to (5, 3)?
19. A cone has volume 1. It is cut parallel to the base at a height ⅓ of the way from the base. What is the area of the truncated cone?
20. Let n be a positive integer less than 50. When you divide n by 9, the remainder is 5. When you divide n by 7, the remainder is 2. What is n?

21. What is the largest sum that results when one of the arrangements of the digits of 2017 is added to one of the of the digits of 2016, if none of the digits 0, 1, or 2 can occupy the same position in both numbers?

22. A carton of six jumbo-sized eggs and six regular-sized eggs weighs 33 oz. If the carton weighs 12 oz and each regular-sized eggs weighs 1.5 oz, how much do three jumbo-sized eggs weigh, in ounces?

23. A right triangle ABC has area 30 with leg AB = 5. What is the side length of the other leg?

24. John's family ordered a 12-slice pizza for dinner. John ate two slices, his brother Jimmy ate half of what John ate, and his brother Jerry ate half of what Jimmy ate. What fraction of the pizza did the three brothers eat?

25. A rectangular frame has a border of 2 inches wide and is 8 inches by 10 inches in total. What is the largest area picture that could be fitted inside the frame, in inches squared?

26. What is the units digit of 7^{81}?

27. How many four digit numbers are there greater than 1000 that use the four digits of 2017?

28. Tom needs to replace the lightbulb in his kitchen located 12 centimeters below his ceiling. The ceiling is 3 meters above the floor, Tom is 1.5 meters tall and has an arm length of 0.4 meters. If he stands on a stool to just reach the bulb, how tall is the stool, in centimeters?

29. Of the 500 balls in a large bag, 80% of the balls are green and 20% are blue. What is the minimum number of blue balls that must be removed such that at least 90% of the bag is made up of green balls?

30. A square and a circle have the same perimeter and circumference, respectively. What is the ratio between the area of the square and the area of the circle?

31. How many digits are in 30 X 30?

32. What is the sum of the angles in a hexagon?

33. How many ways are there to rearrange the words of GAUSS if the two S's cannot be together?

34. For every 4 Hondas sold, 7 Volkswagens are sold. The dealership sells 32 Hondas. How many Volkswagens are sold?

35. A square measures 10 X 10. Then, the square is stretched and compressed by increasing one side by 10% and decreasing the other by 10%. What is the positive difference between the new area and the old area?

36. A three-digit integer contains the digits 2, 0, and 5. What is the probability that it is divisible by 5?

37. If M/N = 0.6, what is (M+N)/(M-N)?

38. The students in Helen's class all took a test. Of the boys who took the test, ⅔ passed. Of the girls in the class, ¾ passed. An equal number of boys and girls passed the test. What is the minimum possible number of students in the class?

39. For how many positive integers n are n/3 and 3n both three-digit positive integers?

40. Evaluate the absolute difference between the quotient and remainder of 48 divided by 7.

Sprint Round Ends

Gauss School and Gauss Math Circle
2017 Gauss Math Tournament
<u>Grade 5-6 (Target Round 20 minutes)</u>

1. Aliens abduct Billy and administer to him a 7 point quiz, each with a right answer and a wrong answer. Billy guesses and has a 1/2 chance of getting each question correct. What is the probability that Billy gets exactly two questions right?

2. A cube is fully inscribed in a sphere of volume 972pi. (the volume of a sphere is given as 4/3pir^3) What is the length of the longest diagonal of this cube, in simplified radical form?

3. What is the shortest distance between the lines $6x + 8y = 10$ and the origin?

4. The probability of rolling an odd number on an unfair die is 4/5. The probability of rolling a prime number on an unfair die is 3/5. The probability of rolling a 2 is 1/9. What is the probability of rolling a one?

5. When the length of a rectangle is increased by 4 and the width is increased by 3, the area goes up by 54. When the width of a rectangle is increased by 4 and the length of a rectangle is increased by 3, the area goes up by 5. What is the perimeter of the rectangle?

6. Consider the sequence

   1, -2, 3, -4, 5, -6, …
   whose nth term is $(-1)^{(n+1)} * n$. What is the average of the first 200 terms of the sequence?

7. How many line segments have both their endpoints located at the vertices of a given cube?
8. If $2^{(1998)} - 2^{(1997)} - 2^{(1996)} + 2^{(1995)} = k * 2^{(1995)}$, what is the value of k?

<u>Target Round Ends</u>

Name: _____   Grade:_____ Division 2

Sprint Round Answers:

1		21	
2		22	
3		23	
4		24	
5		25	
6		26	
7		27	
8		28	
9		29	
10		30	
11		31	
12		32	
13		33	
14		34	
15		35	
16		36	
17		37	
18		38	
19		39	
20		40	

Target Round Answers:

1		5	
2		6	
3		7	
4		8	

G5-6 Answer Keys:

Sprint Round:

1. [-15]
2. [76241]
3. [10]
4. [⅖]
5. [78]
6. [360]
7. [560]
8. [2sqrt(15)]
9. [31]
10. [32]
11. [7]
12. [210]
13. [180]
14. [9]
15. [10]
16. [0.36]
17. [0.84]
18. [56]
19. [19/27]
20. [23]
21. 13321
22. 9
23. 12
24. 7/24
25. 24
26. 7
27. 18
28. 98
29. 56
30. ¼
31. 3
32. 720
33. 36
34. 49
35. 1
36. ¾
37. -4
38. 17
39. 12
40. 12

Target Keys:

1. 21/128
2. 18
3. 1
4. 14/45
5. 10
6. -0.5
7. 28
8. 3