Gauss School and Gauss Math Circle
2017 Gauss Math Tournament
<u>Grade 7-8 (Sprint Round 50 minutes)</u>

1. Compute $5-7(5^2-3^3)^4$.
2. Solve for x: $(2^{4x+8})(4^{2x+3})=8^{2x+6}$
3. What is the sum of the negative integers that satisfy the inequality 2x − 3 ≥ − 11?
4. If five less than three-fourths of an integer is the same as five more than one-eighth of the same integer, what is the integer?
5. If Kenton walks for 60 minutes at the rate of 3 mph and then runs for 15 minutes at the rate of 8 mph, how many miles will he travel?
6. What is the shortest distance between the lines 6x + 8y = 10 and the origin?
7. On each day except the first, the amount of cheese that a cat eats is the sum of all amounts of every ounce of cheese he has previously eaten. Given that on the 567th day, he ate 4578 ounces of cheese, how many ounces did he eat on day 566?
8. A three-digit integer contains the digits 2, 0, and 5. What is the probability that it is divisible by 5?
9. In how many ways can $10 be split among three kids to buy lollipops? (Every kid must have at least one lollipop, which costs $1.
10. There exists sets A and B, such that the total number of elements in set A is twice the total number of elements in set B. Altogether, there are 3011 elements in the union of A and B, and their intersection has 1000 elements. What is the total number of elements in set A?
11. Each term of a sequence is one more than twice the term before it. If the first term is 1, what is the sum of the first 5 terms of the sequence?
12. If $\frac{x}{y}=\frac{4}{7}$ and $\frac{y}{z}=\frac{14}{3}$, what is the value of $\frac{x+y}{z}$? Express your answer as a common fraction.
13. If a fly is buzzing randomly around a room 8 ft long, 12 ft wide and 10 ft high, what is the probability that, at any given time, the fly is within 6 feet of the ceiling? Express your answer as a common fraction.
14. If X and Y are each integers greater than 3 and less than 20, what is the sum of the three possible values of x that satisfy the equation $\frac{x}{y}=\frac{3}{4}$
15. The sum of three consecutive prime numbers is 173. What is the largest of these numbers?
16. The point A(3, 4) is reflected over the x-axis to B. Then B is reflected over the line y = x to C. What is the area of triangle ABC?

17. If a committee of six students is chosen at random from a group of six boys and four girls, what is the probability that the committee contains the same number of boys and girls? Express your answer as a common fraction.
18. Two similar right triangles have areas of 6 square inches and 150 square inches. The length of the hypotenuse of the smaller triangle is 5 inches. What is the sum of the lengths of the legs of the larger triangle?
19. Teacher Helen normally spends a half-hour driving to class. When her average speed is ten miles per hour less than usual, the trip takes ten minutes longer. How many miles does she drive to class?
20. The probability of rolling an odd number on an unfair die is 4/5. The probability of rolling a prime number on the same unfair die is 3/5. The probability of rolling a 2 is 1/9. What's the probability of rolling a 1?
21. A pair of positive integers (x, y) satisfies the equation $31x + 29y = 1135$. What is $x + y$?
22. Two cylindrical cans have the same volume. The height of one can is triple the height of the other. If the radius of the narrower can is 12 units, how many units are in the length of the radius of the wider can? Express your answer in simplest radical form.
23. How many ways can $25 be made out of dimes, nickels and pennies?
24. A calculator can only do two functions: subtract one and square. If we start with the number three, what is the minimum amount of steps needed to get to 2016*2018?
25. Which integer is closest to $\frac{1}{2}(\sqrt[3]{829} + \log_{10} 829)$?
26. Equilateral triangle ABC is inscribed in a unit circle. Find the radius of the largest circle which is tangent to both BC and the circle.
27. If $X^2 + 5X + 7 = 0$ let a and b be the two solutions. Find $a + \frac{1}{a^2} + \frac{1}{b^2} + \frac{7}{a}$.
28. What's the greatest positive integer n such that 3^n is a factor of 200?
29. What's the area of a triangle in square cm with side length of 14 cm, 13cm and 15cm, respectively?
30. Find the sum of all solutions to the equation of $\frac{x^2}{x-4} = 2x + 9 + \frac{11}{x-4}$
31. You have 3 blue tokens. Every day, you either gain a red token, or double the amount of blue tokens you have. What is the sum of the distinct amounts of tokens you could have at the end of 3 days?
32. Consider the sequence 1, -2, 3, -4, 5, -6………, whose Nth term is $-1^{N+1} * N$. What's the average of the first 200 terms of the sequence?
33. A right triangle has side lengths in an arithmetic progression. If the inscribed circle of the triangle has a radius r=10, find the area of this triangle.
34. Compute the smallest positive integer with exactly 26 positive composite factors, i.e. factors other than 1 or a prime number.

35. Ana wants to make a cylindrical can with the maximum volume. If she is allowed only 100π square inches of lateral surface area, and radius and height are both integral amounts of inches, find the volume of this cylindrical can.
36. A triangle has a side of length 6 cm, a side of length 8 cm and a right angle. What is the shortest possible length of the remaining side of the triangle? Express your answer as a decimal to the nearest hundredth.
37. If 2^(1998) - 2^(1997) - 2^(1996) + 2^(1995) = k * 2^(1995), what is the value of k?
38. A triangle ABC has altitude AD and orthocenter H. Given that BD=20, CD=35, and $\frac{AB}{AC}=\frac{20}{17}$, find the value of AD*HD.
39. An ant starts at one vertex of a tetrahedron. At any given moment, it has equal probability of going to any of the other three vertices. Find the probability it is back at the first vertex after 6 moves.
40. A rhombus of side length s has the property that there is a point on its longer diagonal such that the distance from that point to the vertices are 1, 1, 1, and s. what is the value of s?

<u>Sprint Round Ends</u>

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1. How many ordered tripled of three prime numbers exist for which for sum of the members of the triple is 24?
2. Given that M and Nare digits, what the sum of the values for M and N which yield the greatest six-digit number 5M5,62N, that is divisible by 44.
3. An ant is standing at the origin of a coordinate grid. The ant will take four steps, each 1 unit in length. Each step taken is either forward, backward, right or left, chosen at random. What is the probability that the ant's fourth step places the ant back at the origin? Express your answer as a common fraction.
4. If X < 0 and y < 0, solve the system of equations

$$x^2 + xy + x = 14$$

$$y^2 + xy + y = 28$$

   Write your answer as an ordered pair (x,y)

5. Two people, A and B, get on two very long escalators going down at the same speed. A walks twice as fast as B does. When A gets to the bottom of the escalator, he has taken 648 steps. When B gets to the bottom of the escalator, he has taken 486 steps. How many steps of the escalator are visible at a time?
6. Compute the least prime p such that p-1 equals the difference of the squares of two positive multiplies of 4.
7. Each side of triangle ABC has length 2. A circle with center at A and radisu 1 cuts AB at M. A tangent to the circle from B and lying outside the triangle meets the circle at P. What's the area of the regions bound by BP, BM and the minor arc MP?
8. A semicircle is inscribed in a quadrilateral *ABCD* in such a way that the midpoint of *BC* coincides with the center of the semicircle, and the diameter of the semicircle lies along a portion of *BC*. If *AB* = 4 and *CD* = 5, what is BC?

Target Round Ends

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Name: ______________________________ Grade:________________________ Division 3

Sprint Round Answers:

1		21	
2		22	
3		23	
4		24	
5		25	
6		26	
7		27	
8		28	
9		29	
10		30	
11		31	
12		32	
13		33	
14		34	
15		35	

16		36	
17		37	
18		38	
19		39	
20		40	

Target Round Answers:

1		5	
2		6	
3		7	
4		8	

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41. (-107)
42. (2)
43. (−10)
44. (16)
45. (3)
46. (1)
47. (2289)
48. (3/4)
49. (36)
50. (2674)
51. (57)
52. (22/7)
53. (3/5)
54. (27)
55. (61)
56. (28)
57. (8/21)
58. (60)
59. (20 miles)
60. (14/25)
61. (37)
62. ($12\sqrt{3}$)
63. (63001)
64. (19)
65. (6)
66. (3/4)
67. (52)
68. (97)
69. (84 cm^2)
70. (answer: -5)
71. (58)
72. (-0.5)
73. (600)
74. (720)
75. (2500π)
76. (5.29 cm)
77. (3)

78. (700)
79. (61/243)
80. ($\frac{1+\sqrt{5}}{2}$)

Grade 7-8 (Target Round 20 minutes)

9. (15)
10. (17)
11. (9/64)

4. ($-\frac{7}{2}, \frac{14}{2}$))

5. (972)

6. (113)
7. (($\frac{\sqrt{2}}{2} - \frac{\pi}{6}$))
8. ($4\sqrt{5}$)